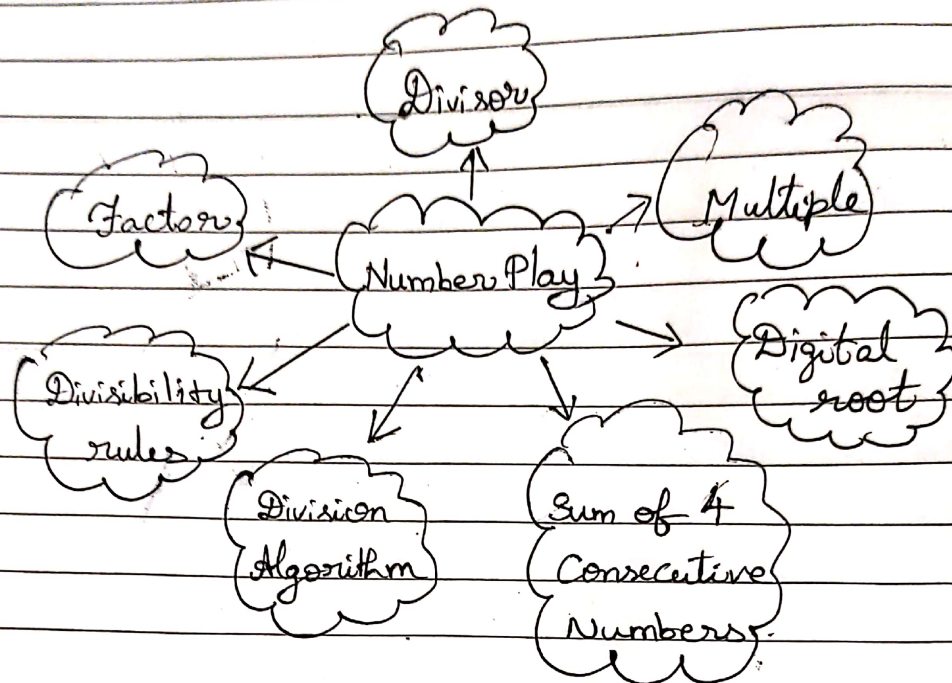


Chapter 5. Number Play

* Mindmap:



* Divisor =

A divisor is a number that divides another number exactly, with no remainder.

* Multiple:

A multiple is a number that is the result of multiplying a number by an integer.

* Factor:

A number that divides another number exactly, then the divisor is called a factor of the dividend.

* Multiple:

A multiple is a number that is the result

of multiplying a number by an integer.

* The division algorithm: [Division Theorem]:
[Euclid Division Lemma]:

Any 2 integers a and b where b is a non zero integer, there exist unique integers q (quotient) and r (remainder) such that the following condition is met: $a = bq + r$.

* Divisibility Rules:

A number is divisible by	
2	if its units digit is 0, 2, 4, 6, 8
3	sum of the digits is divisible by 3
4	if the last 2 digits '00' or divisible by 4.
5	if its units digit is 0 (or) 5.
6	if it is divisible by both 2 and 3.
7	double the last digit and subtract from the remaining digits. (or) The difference of the number of its thousands and the remaining part of the given number is divisible by 7.
8	The last 3 digits of the given number is divisible by 8.
9	sum of the digits is divisible by 9.
10	if the last digit is 0.

11	if the difference of the sum of the digits at odd places and the sum of its digits at even places, is either '0' or divisible by 11.
12	if it is divisible by both 4 and 3.
14	if it is divisible by both 2 and 7.
15	if it is divisible by both 3 and 5.
16	if the number formed by the last 4 digits is divisible by 16.
24	if it is divisible by both 3 and 8.
40	if it is divisible by both 5 and 8.
80	if it is divisible by both 5 and 16.

Digits in disguise:

- Digits in disguise also known as "Cryptarithm".
- A cryptarithm is a mathematical puzzle where letters or symbols replace hidden digits from 0 to 9.
- Each letter represents a single unique digit throughout the puzzle.
- The first digit of a multidigit number can never be zero.
- Use basic addition, multiplication and carry over rules to test values.

* Digital roots:

The digits in the number is add up till it gets single digit is called digital roots.

Eg: Find the digital root of 7694

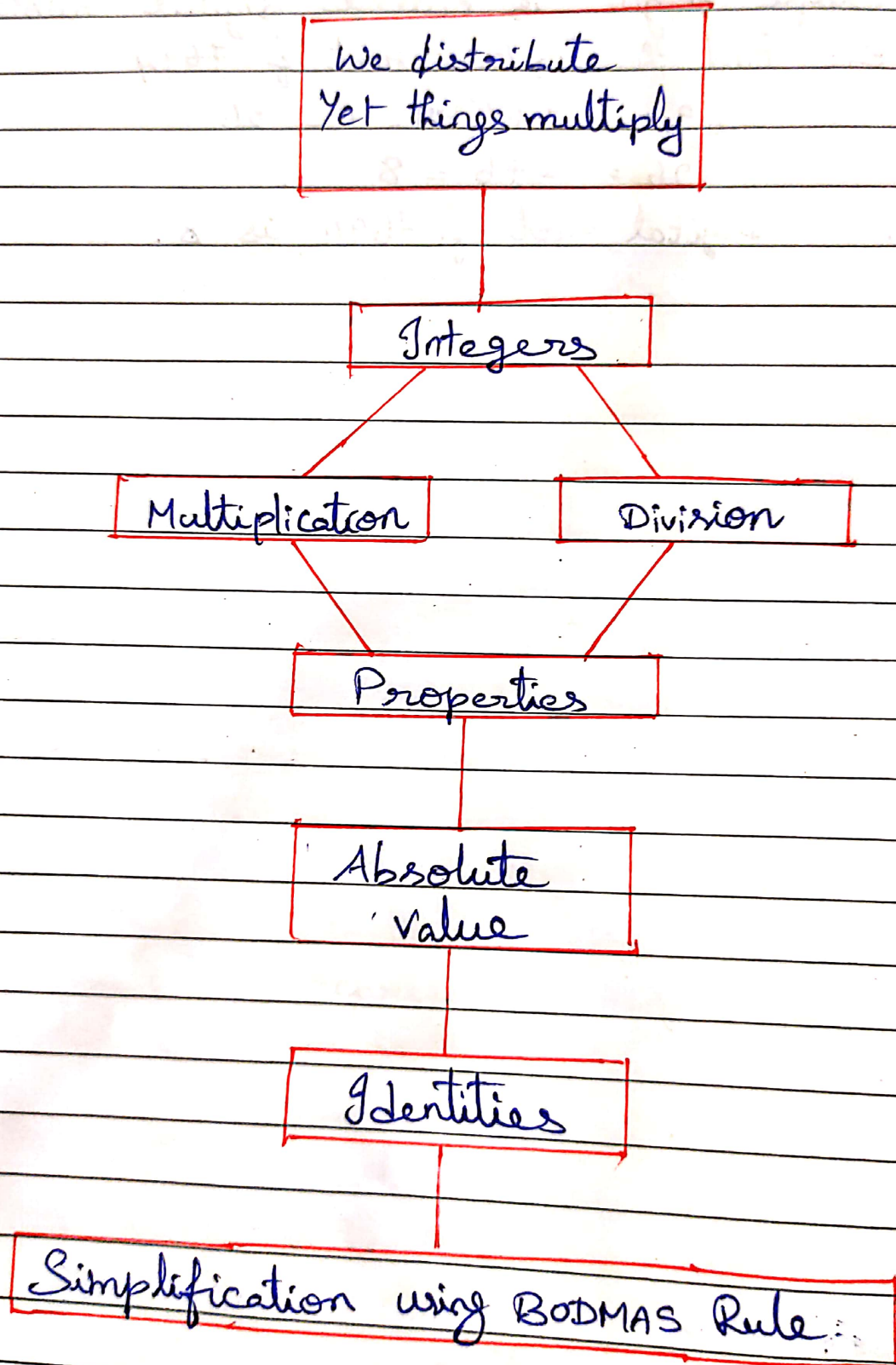
$$7694 = 7 + 6 + 9 + 4 = 26$$

$$26 = 2 + 6 = 8$$

Digital root of 7694 is 8.

chap 6. We distribute, yet things multiply.

* Mindmap:



* A combination of constants and variables connected by the signs of fundamental operations (+, -, ×, ÷) is called an Algebraic expression.

* Multiplication of Integers.

- $(+) \times (+) = +$
- $(+) \times (-) = -$
- $(-) \times (+) = -$
- $(-) \times (-) = +$
- odd number of negative integers, gives a negative integer.
- Even number of negative integers gives a positive integer.

* Properties of Multiplication of Integers:

- Closure $\rightarrow a \times b$ is also an integer.
- Commutative $\rightarrow a \times b = b \times a$
- Associative $\rightarrow a \times (b \times c) = (a \times b) \times c$
- 1 is called the multiplicative identity (or) Identity element.
- The product of an integer and zero is zero.
- Distributive property of multiplication over addition and subtraction.

$$a \times (b + c) = (a \times b) + (a \times c)$$

$$a \times (b - c) = (a \times b) - (a \times c)$$

* Division of Integers:

- If ^{the} dividend and divisor have the same sign, the quotient is positive.
- If the dividend and divisor have the different sign, the quotient is negative.

* Properties of division of Integers:

- closure $\rightarrow a \div b$ is not an integer.
- commutative $\rightarrow a \div b \neq b \div a$
- Associative $\rightarrow a \div (b \div c) \neq (a \div b) \div c$
- Any integer divided by 1 gives the quotient is the integer itself.

$$a \div 1 = a$$

- Division of 0 by any non zero integer is zero.

$$0 \div a = 0$$

* Simplification of Numerical expressions:

- The order of operations used to simplify an algebraic expression is BODMAS rule.
- The order of working is first the terms within the round brackets $()$, curly brackets $\{ \}$, square brackets $[]$

* Absolute value of an integer:

- It is the numerical value of the integer regardless of its sign.
- It is denoted as $|a|$ and defined as $|a| = a$ if $a \geq 0$.

* Mind the mistake, Mend the mistake:

- Distribute the number outside the bracket

to every term inside the bracket.

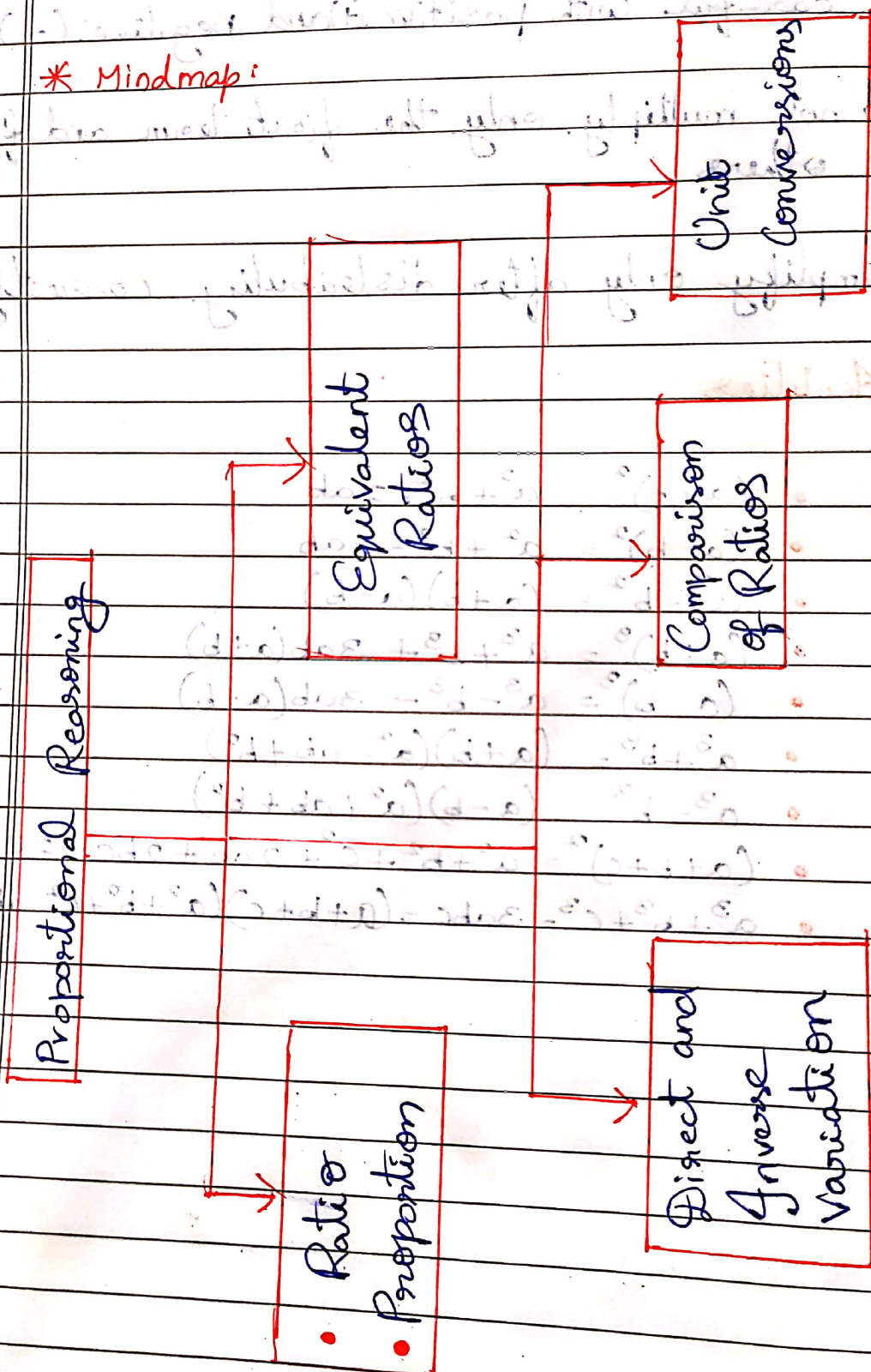
- Be careful with positive (+) and negative (-) signs.
- Do not multiply only the first term and forget the others
- Simplify only after distributing correctly.

* Identities :

- $(a+b)^2 = a^2 + b^2 + 2ab$
- $(a-b)^2 = a^2 + b^2 - 2ab$
- $a^2 - b^2 = (a+b)(a-b)$
- $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Chapter 7: Proportional Reasoning - 1

* Mindmap:



* Similar figures:

Similar figures have the same shape, ratios and proportions, but not same dimensions.

* Ratio:

A ratio is the comparison of two quantities of the same kind can be represented as $x:y$ (or) $\frac{x}{y}$
 x is called as antecedent.
 y is called as consequent.

* Properties of Ratio:

- Ratio is a fraction and has no unit.
- Ratio should be expressed in the lowest terms.
- Ratio can be simplified by dividing both terms by their HCF.

* Equivalent Ratio:

Equivalent ratios can be obtained by multiplying (or) dividing both the antecedent and the consequent by the same non zero number.

* Proportion:

- If two ratios are equal, then four numbers forming the two ratios in order are said to be in proportion.

$a:b = c:d$, then a, b, c, d are in proportion

- If 4 numbers are in proportion, ^{then} the product of extremes = product of means.

$$ad = bc$$

* Mean proportional :

The mean proportional between 2 numbers is the middle term in a continued proportion.

$$a : x = x : b$$

$$ab = x^2$$

$$x = \sqrt{ab}$$

$$(or) b = \frac{x^2}{a}$$

* Direct Variation :

If 2 variables x and y vary in such a way that their ratio $\frac{x}{y}$ is a constant,

then x and y are said to be in direct variation.

$$\frac{x}{y} = k \quad (or) \quad x = ky$$

eg: Number of days worked and amount of earnings.

* Inverse Proportion :

If 2 variables x and y vary in such a way that $xy = k$, (constant), then x and y are said to be in inverse proportion.

$$xy = k$$

eg: Number of people doing a work and time taken to complete the work.

* Composition of ratios:

• Compounded ratio:

The compounded ratio of $a:b$, $c:d$, $e:f$ can be represented as $ace : bdf$

• Reciprocal ratio:

the reciprocal ratio of $a:b$ is $b:a$

• Duplicate ratio:

the duplicate ratio of $a:b$ is $a^2:b^2$

• Triplicate ratio:

the triplicate ratio of $a:b$ is $a^3:b^3$

• Subduplicate ratio:

the subduplicate ratio of $a:b$ is $\sqrt{a}:\sqrt{b}$

• Subtriplicate ratio:

the subtriplicate ratio of $a:b$ is $\sqrt[3]{a}:\sqrt[3]{b}$

* Unit conversions:

Length: $1 \text{ inch} = 2.54 \text{ cm}$, $1 \text{ ft} = 12 \text{ in}$, $1 \text{ km} = 1000 \text{ m}$

Area: $1 \text{ m}^2 = 10^4 \text{ cm}^2$ (because 100^2)

Volume: $1 \text{ m}^3 = 10^6 \text{ cm}^3$; $1 \text{ L} = 1000 \text{ mL} = 1000 \text{ cm}^3$

Mass: $1 \text{ kg} = 1000 \text{ g}$; $1 \text{ lb} = 0.453592 \text{ kg}$

Time: $1 \text{ hr} = 3600 \text{ sec}$

Speed: $1 \text{ m s}^{-1} = 3.6 \text{ km h}^{-1}$

Pressure: $1 \text{ atm} = 101325 \text{ Pa}$

Energy: $1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$; $1 \text{ cal} \approx 4.184 \text{ J}$

Power: $1 \text{ W} = 1 \text{ J s}^{-1}$

* Temperature :

$$\text{Celsius} \leftrightarrow \text{Kelvin} : K = ^\circ C + 273.15$$

$$\text{Celsius} \leftrightarrow \text{Fahrenheit} : ^\circ F = \frac{9}{5} ^\circ C + 32$$

$$^\circ C = \frac{5}{9} (^\circ F - 32)$$